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THERMOELASTIC ANALYSIS OF CROSS-PLY LAMINATED CIRCULAR CYLINDRICAL SHELLS

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Abstract— Thermal deformations and stresses in cross-ply laminated circular cylindrical shells are investigated. The state space approach is used to solve exactly the thermoelastic governing equations of the third-order, first-order and classical theories for arbitrary boundary conditions. To illustrate the thermoelastic behavior, exact analytical solutions for deflections and stresses are obtained for laminated circular cylindrical shells undergoing uniform and linearly varying temperature fields through the thickness while having sinusoidal distribution of thermal loading. Copyright © 1996 Elsevier Science Ltd

INTRODUCTION

Composite materials have found increasing application in many engineering structures. The applications range from the design of machines to high performance aircrafts. The strength and light-weight requirements of space structures have created considerable interest in the study of laminated circular cylindrical shells. It is further fuelled by the fact that the classical lamination shell theory based on the Love–Kirchhoff assumptions is adequate to predict the gross behavior of thin laminates. When the structures are rather thick or when they exhibit high anisotropy ratios, the transverse shear deformation effect has to be incorporated. In such cases more refined theories are needed.

The analysis of the thermal response of plate and shell structures has been the subject of significant research interest in recent years [see Pell (1946), Stavsky (1963), Wu (1978), Kalam and Tauchert (1978), Reddy and Hsu (1980), Wu and Tauchert (1980a, b), Hyer and Cooper (1986), Kardomateas (1989), Khdeir and Reddy (1991), Khdeir *et al.* (1992)]. The problem of thermal bending of anisotropic plates was first studied by Pell (1946), who derived the equations governing the transverse deflection of a thin plate. Generalization of Pell's work to heterogeneous plates subjected to arbitrary three-dimensional temperature distribution is due to Stavsky (1963), who obtained the deformation and stresses in a thin rectangular plate, simply supported along two infinitely long edges and subject to uniform heating. Thermally induced deformations and stress resultants in symmetric laminated plates are analyzed by Wu and Tauchert (1980a). The method of Levy is used to study the transverse bending of specially orthotropic laminates having two opposite edges simply supported and subject to a temperature distribution using the classical laminate theory. A finite element formulation of the equations of the first-order theory of anisotropic composite plates subjected to thermal and mechanical loadings is presented by Reddy and Hsu (1980). Formulations and solutions for the thermal stresses in orthotropic cylinders have been presented by Kalam and Tauchert (1978) due to a steady-state plane temperature distribution. A linear elasticity solution for determining the response of cross-ply composite tubes subjected to a circumferential temperature gradient of the form $\Delta T_0 + \Delta T_1 \cos(\theta)$ is presented by Hyer and Cooper (1986). Temperature-independent material properties are assumed and displacement approach is used. The same approach is used by Kardomateas (1989) to find stresses and displacements in an orthotropic, hollow circular cylinder, due to an imposed constant temperature on the one surface and heat convection into a medium of a different constant temperature at the outer surface.

Analytical solutions are proposed by many investigators for the case of simply supported edge conditions with different formulations [see Wu and Tauchert (1980b), Reddy and Hsu (1980)]. This case is often used in the literature as a test case. Thermoelastic coupling of the governing equations of laminated plates is investigated in Wu and Tauchert (1980b) for simply supported edge conditions and in [Khdeir and Reddy (1991), Khdeir *et al.* (1992)] for cross-ply laminated plates and shallow shells whose two opposite edges are simply supported and the remaining ones have arbitrary combinations of boundary conditions using refined theories. For other types of boundary conditions and for more arrangements of lamina, approximate methods such as the Rayleigh–Ritz technique and finite element are used in Wu (1978) and Reddy and Hsu (1980), respectively.

The objectives of this paper are to offer an exact solution based on the state space approach to analyze the thermal response of cross-ply laminated circular cylindrical shells for arbitrary boundary conditions and to establish the correlation between classical and refined shell theories.

GOVERNING EQUATIONS

The governing equations of the third-order theory (HSDT) of Reddy and Liu [see Reddy and Liu (1985), Khdeir *et al.* (1989)] as applied to a circular cylindrical shell of length L, radius R and thickness h are

$$\frac{\partial N_1}{\partial x_1}+\frac{\partial N_6}{\partial x_2}=0$$

$$\frac{\partial N_6}{\partial x_1}+\frac{\partial N_2}{\partial x_2}=0$$

$$\frac{\partial Q_1}{\partial x_1}+\frac{\partial Q_2}{\partial x_2}-n_1\left(\frac{\partial K_1}{\partial x_1}+\frac{\partial K_2}{\partial x_2}\right)+n_2\left(\frac{\partial^2 P_1}{\partial x_1^2}+\frac{\partial^2 P_2}{\partial x_2^2}+2\frac{\partial^2 P_6}{\partial x_1\,\partial x_2}\right)-\frac{N_2}{R}+q=0$$

$$\frac{\partial M_1}{\partial x_1}+\frac{\partial M_6}{\partial x_2}-Q_1+n_1K_1-n_2\left(\frac{\partial P_1}{\partial x_1}+\frac{\partial P_6}{\partial x_2}\right)=0$$

$$\frac{\partial M_6}{\partial x_1}+\frac{\partial M_2}{\partial x_2}-Q_2+n_1K_2-n_2\left(\frac{\partial P_6}{\partial x_1}+\frac{\partial P_2}{\partial x_2}\right)=0 \tag{1}$$

where $n_1 = 4/h^2$ and $n_2 = n_1/3$. Here (u, v, w) are the displacements along the axes x_1 (axial), x_2 (circumferential) and ζ (radial), respectively; ϕ_1 and ϕ_2 are the rotations about the x_2 and x_1-axes, q is the distributed transverse mechanical load, and N_i, M_i, ... are the stress resultants

$$(N_i, M_i, P_i)=\sum_{k=1}^{N}\int_{\zeta_{k-1}}^{\zeta_k}\sigma_i^{(k)}(1,\zeta,\zeta^3)\,\mathrm{d}\zeta \quad (i=1,2,6)$$

$$(Q_1, K_1)=\sum_{k=1}^{N}\int_{\zeta_{k-1}}^{\zeta_k}\sigma_5^{(k)}(1,\zeta^2)\,\mathrm{d}\zeta$$

$$(Q_2, K_2)=\sum_{k=1}^{N}\int_{\zeta_{k-1}}^{\zeta_k}\sigma_4^{(k)}(1,\zeta^2)\,\mathrm{d}\zeta. \tag{2}$$

The stress–strain relations for the kth lamina in the laminate coordinate are given by

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_6\\ \sigma_4\\ \sigma_5\end{Bmatrix}_{(k)}=\begin{bmatrix}Q_{11} & Q_{12} & 0 & 0 & 0\\ & Q_{22} & 0 & 0 & 0\\ & & Q_{66} & 0 & 0\\ & \text{symm.} & & Q_{44} & 0\\ & & & & Q_{55}\end{bmatrix}_{(k)}\begin{Bmatrix}\varepsilon_1-\alpha_{11}\,\Delta T\\ \varepsilon_2-\alpha_{22}\,\Delta T\\ \varepsilon_6\\ \varepsilon_4\\ \varepsilon_5\end{Bmatrix}_{(k)} \tag{3}$$

where $Q_{ij}^{(k)}$ are the material constants of the kth lamina in the laminate coordinate system and $\alpha_{11}^{(k)}$ and $\alpha_{22}^{(k)}$ are the coefficients of linear thermal expansion for layer k in the laminate coordinate; ΔT denotes the temperature rise in the laminate and is given by:

$$\Delta T = T_0(x_1, x_2) + \zeta T_1(x_1, x_2). \tag{4}$$

The linear strain-displacement relations are:

$$\begin{aligned}
\varepsilon_1 &= \varepsilon_1^0 + \zeta(\kappa_1^0 + \zeta^2 \kappa_1^2) \\
\varepsilon_2 &= \varepsilon_2^0 + \zeta(\kappa_2^0 + \zeta^2 \kappa_2^2) \\
\varepsilon_4 &= \varepsilon_4^0 + \zeta^2 \kappa_4^1 \\
\varepsilon_5 &= \varepsilon_5^0 + \zeta^2 \kappa_5^1 \\
\varepsilon_6 &= \varepsilon_6^0 + \zeta(\kappa_6^0 + \zeta^2 \kappa_6^2),
\end{aligned} \tag{5}$$

where

$$\begin{aligned}
&\varepsilon_1^0 = \frac{\partial u}{\partial x_1}, \quad \kappa_1^0 = \frac{\partial \phi_1}{\partial x_1}, \quad \kappa_1^2 = -n_2\left(\frac{\partial \phi_1}{\partial x_1} + \frac{\partial^2 w}{\partial x_1^2}\right), \\
&\varepsilon_2^0 = \frac{\partial v}{\partial x_2} + \frac{w}{R}, \quad \kappa_2^0 = \frac{\partial \phi_2}{\partial x_2}, \quad \kappa_2^2 = -n_2\left(\frac{\partial \phi_2}{\partial x_2} + \frac{\partial^2 w}{\partial x_2^2}\right), \\
&\varepsilon_4^0 = \phi_2 + \frac{\partial w}{\partial x_2}, \quad \kappa_4^1 = -n_1\left(\phi_2 + \frac{\partial w}{\partial x_2}\right), \\
&\varepsilon_5^0 = \phi_1 + \frac{\partial w}{\partial x_1}, \quad \kappa_5^1 = -n_1\left(\phi_1 + \frac{\partial w}{\partial x_1}\right), \\
&\varepsilon_6^0 = \frac{\partial v}{\partial x_1} + \frac{\partial u}{\partial x_2}, \quad \kappa_6^0 = \frac{\partial \phi_2}{\partial x_1} + \frac{\partial \phi_1}{\partial x_2}, \quad \kappa_6^2 = -n_2\left(\frac{\partial \phi_2}{\partial x_1} + \frac{\partial \phi_1}{\partial x_2} + 2\frac{\partial^2 w}{\partial x_1 \partial x_2}\right).
\end{aligned} \tag{6}$$

The resultants are related to the total strains by:

$$\begin{aligned}
N_i &= A_{ij}\varepsilon_j^0 + B_{ij}\kappa_j^0 + E_{ij}\kappa_j^2 - N_i^T \\
M_i &= B_{ij}\varepsilon_j^0 + D_{ij}\kappa_j^0 + F_{ij}\kappa_j^2 - M_i^T, \quad (i, j = 1, 2, 6) \\
P_i &= E_{ij}\varepsilon_j^0 + F_{ij}\kappa_j^0 + H_{ij}\kappa_j^2 - P_i^T
\end{aligned} \tag{7}$$

$$\begin{aligned}
Q_2 &= A_{4j}\varepsilon_j^0 + D_{4j}\kappa_j^1 \\
Q_1 &= A_{5j}\varepsilon_j^0 + D_{5j}\kappa_j^1 \quad (j = 4, 5) \\
K_2 &= D_{4j}\varepsilon_j^0 + F_{4j}\kappa_j^1 \\
K_1 &= D_{5j}\varepsilon_j^0 + F_{5j}\kappa_j^1
\end{aligned} \tag{8}$$

where A_{ij}, B_{ij}, etc. are the laminate stiffnesses,

$$(A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}) = \sum_{k=1}^{N} \int_{\zeta_{k-1}}^{\zeta_k} Q_{ij}^{(k)}(1, \zeta, \zeta^2, \zeta^3, \zeta^4, \zeta^6)\, d\zeta \tag{9}$$

for $i, j = 1, 2, 4, 5, 6$, and the thermal forces and moments are defined by

$$\begin{Bmatrix} N_1^T, M_1^T, P_1^T \\ N_2^T, M_2^T, P_2^T \end{Bmatrix} = \sum_{k=1}^{N} \int_{\zeta_{k-1}}^{\zeta_k} \begin{bmatrix} Q_{11}^{(k)} & Q_{12}^{(k)} \\ Q_{12}^{(k)} & Q_{22}^{(k)} \end{bmatrix} \begin{Bmatrix} \alpha_{11}^{(k)} \\ \alpha_{22}^{(k)} \end{Bmatrix} (1, \zeta, \zeta^3)\, \Delta T \, \mathrm{d}\zeta. \tag{10}$$

The governing equations of the first-order shear deformation theory (FSDT) can be deduced from those of the third-order theory (HSDT) by setting $n_1 = n_2 = 0$, and the classical theory (CST) is obtained from the first-order theory by setting

$$\phi_i = -\frac{\partial w}{\partial x_i} \quad (i = 1, 2). \tag{11}$$

ANALYTICAL FORMULATION

The state space approach is used to analyze the thermal bending of cross-ply laminated composite circular cylindrical shells for all boundary conditions. The edges ($x_1 = \pm L/2$) may have arbitrary combinations of free, clamped and simply supported edge conditions. A sinusoidal distribution of the thermal loadings will be considered [see Reddy and Hsu (1980)], which for the present case takes the form:

$$\begin{Bmatrix} T_0 \\ T_1 \end{Bmatrix} = \begin{Bmatrix} \bar{T}_0 \\ \bar{T}_1 \end{Bmatrix} \cos \alpha x_1 \cos \beta x_2. \tag{12}$$

The displacement quantities will be expressed as products of undetermined functions and known trigonometric functions so as to satisfy the governing eqns (1) and loading conditions (12):

$$\begin{aligned} u(x_1, x_2) &= U(x_1) \cos \beta x_2 \\ v(x_1, x_2) &= V(x_1) \sin \beta x_2 \\ w(x_1, x_2) &= W(x_1) \cos \beta x_2 \\ \phi_1(x_1, x_2) &= \Phi_1(x_1) \cos \beta x_2 \\ \phi_2(x_1, x_2) &= \Phi_2(x_1) \sin \beta x_2 \end{aligned} \tag{13}$$

where $\alpha = \pi/L$ and $\beta = 1/R$; the mechanical loading q is set to zero throughout the analysis.

Substitution of eqns (12) and (13) into the governing equations of the HSDT theory results in the following system of equations

$$\begin{aligned} U'' &= c_1 U + c_2 V' + c_3 W' + c_4 W''' + c_5 \Phi_1 + c_6 \Phi_2' + g_1 \sin \alpha x_1 \\ V'' &= c_7 U' + c_8 V + c_9 W + c_{10} W'' + c_{11} \Phi_1' + c_{12} \Phi_2 + g_2 \cos \alpha x_1 \\ W'''' &= c_{13} U' + c_{14} V + c_{15} W + c_{16} W'' + c_{17} \Phi_1' + c_{18} \Phi_2 + g_3 \cos \alpha x_1 \\ \Phi_1'' &= c_{19} U + c_{20} V' + c_{21} W' + c_{22} W''' + c_{23} \Phi_1 + c_{24} \Phi_2' + g_4 \sin \alpha x_1 \\ \Phi_2'' &= c_{25} U' + c_{26} V + c_{27} W + c_{28} W'' + c_{29} \Phi_1' + c_{30} \Phi_2 + g_5 \cos \alpha x_1 \end{aligned} \tag{14}$$

where a prime on a quantity denotes the derivative with respect to x_1. The coefficients g_i in eqn (14) are defined as follows:

$$\begin{aligned} g_1 &= (e_3 f_4 - e_{30} f_1)/e_0 \\ g_2 &= (e_{12} f_5 - e_{39} f_2)/c_0 \\ g_4 &= (e_{28} f_1 - e_1 f_4)/e_0 \\ g_5 &= (e_{37} f_2 - e_{10} f_5)/c_0 \\ g_3 &= a_0 (g_2 a_1 + g_5 a_2 - f_3 + \alpha e_{21} g_1 + \alpha e_{23} g_4) \end{aligned} \tag{15}$$

where

$$
\begin{aligned}
f_1 &= -\alpha(L_1\bar{T}_0+L_2\bar{T}_1)\\
f_2 &= -\beta(L_3\bar{T}_0+L_4\bar{T}_1)\\
f_3 &= n_2[-\alpha^2(L_7\bar{T}_0+L_8\bar{T}_1)-\beta^2(L_9\bar{T}_0+L_{10}\bar{T}_1)]-(L_3\bar{T}_0+L_4\bar{T}_1)/R\\
f_4 &= -\alpha(L_2\bar{T}_0+L_5\bar{T}_1)+n_2\alpha(L_7\bar{T}_0+L_8\bar{T}_1)\\
f_5 &= -\beta(L_4\bar{T}_0+L_6\bar{T}_1)+n_2\beta(L_9\bar{T}_0+L_{10}\bar{T}_1).
\end{aligned}
\tag{16}
$$

L_i appearing in eqn (16) are defined by:

$$
\begin{Bmatrix} L_1, L_2, L_5, L_7, L_8 \\ L_3, L_4, L_6, L_9, L_{10} \end{Bmatrix} = \sum_{k=1}^{N} \int_{\zeta_{k-1}}^{\zeta_k} \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12} & Q_{22} \end{bmatrix}_{(k)} \begin{Bmatrix} \alpha_{11} \\ \alpha_{22} \end{Bmatrix}_{(k)} (1,\zeta,\zeta^2,\zeta^3,\zeta^4)\,\mathrm{d}\zeta. \tag{17}
$$

The coefficients c_i, e_i and a_i in eqns (14), (15), and (16) are presented in Khdeir (1995).

In order to reduce the system of eqns (14) to a state form, the components of the state vector $\{\mathbf{Y}(x_1)\}$ [see Brogan (1985), Franklin (1968)] associated with HSDT theory are defined as

$$
\begin{aligned}
&Y_1 = U,\ Y_2 = U',\ Y_3 = V,\ Y_4 = V',\ Y_5 = W,\ Y_6 = W',\\
&Y_7 = W'',\ Y_8 = W''',\ Y_9 = \Phi_1,\ Y_{10} = \Phi_1',\ Y_{11} = \Phi_2,\ Y_{12} = \Phi_2'.
\end{aligned}
\tag{18}
$$

Using (18), and after some algebraic manipulation, the system of eqns (14) may be expressed in the form

$$\{\mathbf{Y}'\} = [\mathbf{A}]\{\mathbf{Y}\}+\{\mathbf{r}\} \tag{19}$$

where the matrix [**A**] will be defined as:

$$
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
c_1 & 0 & 0 & c_2 & 0 & c_3 & 0 & c_4 & c_5 & 0 & 0 & c_6\\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & c_7 & c_8 & 0 & c_9 & 0 & c_{10} & 0 & 0 & c_{11} & c_{12} & 0\\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\
0 & c_{13} & c_{14} & 0 & c_{15} & 0 & c_{16} & 0 & 0 & c_{17} & c_{18} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\
c_{19} & 0 & 0 & c_{20} & 0 & c_{21} & 0 & c_{22} & c_{23} & 0 & 0 & c_{24}\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\
0 & c_{25} & c_{26} & 0 & c_{27} & 0 & c_{28} & 0 & 0 & c_{29} & c_{30} & 0
\end{bmatrix}
\tag{20}
$$

The elements of the load vector $\{\mathbf{r}\}$ are

$$\{\mathbf{r}\}^{\mathrm{T}} = \{0, g_1\sin\alpha x_1, 0, g_2\cos\alpha x_1, 0,0,0, g_3\cos\alpha x_1, 0, g_4\sin\alpha x_1, 0, g_5\cos\alpha x_1\}. \tag{21}$$

The solution to eqn (19) is given by [see Brogan (1985), Franklin (1968)]:

$$\{\mathbf{Y}(x_1)\} = [\mathbf{D}]\begin{bmatrix} e^{\lambda_1 x_1} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & e^{\lambda_{12} x_1} \end{bmatrix}\{\mathbf{k}\} + \int [\mathbf{D}]\begin{bmatrix} e^{\lambda_1 (x_1-\eta)} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & e^{\lambda_{12}(x_1-\eta)} \end{bmatrix}[\mathbf{D}]^{-1}\{\mathbf{r}(\eta)\}\,\mathrm{d}\eta. \quad (22)$$

Here $\{\mathbf{k}\}$ is a constant column vector to be determined from the edge conditions, where λ_i are the distinct eigenvalues of the matrix $[\mathbf{A}]$ while $[\mathbf{D}]$ denotes the matrix of eignevectors of $[\mathbf{A}]$.

The following boundary conditions for simply supported (S), clamped (C) and free (F) at the edges $x_1 = \pm L/2$ for the HSDT theory are used in the analysis:

$$S: v = w = \phi_2 = N_1 = M_1 = P_1 = 0$$

$$C: u = v = w = \phi_1 = \phi_2 = \frac{\partial w}{\partial x_1} = 0$$

$$F: N_1 = M_1 = P_1 = N_6 = M_6 - n_2 P_6 = Q_1 - n_1 K_1 + n_2\left(\frac{\partial P_1}{\partial x_1} + 2\frac{\partial P_6}{\partial x_2}\right) = 0. \quad (23)$$

Using (18) and (22), the displacement quantities will be found from eqn (13).

A similar procedure has been followed to find the thermal response of cross-ply circular cylinder shells using FSDT and CST theories. For the sake of conciseness it is not reported in this paper.

NUMERICAL RESULTS AND DISCUSSION

To demonstrate the method developed in the present study, numerical results are displayed to show the variation of thermal deformations and stresses with the variation of loading, geometry, lamination, theory, number of layers and boundary conditions. The numerical results have been obtained analytically and exactly by using eqns (22), (18), (13) and (3) for the desired boundary conditions defined in (23). Symmetric and antisymmetric cross-ply lamination schemes have been used. It was assumed that the thickness and the material for all laminae are the same, having the following characteristics:

$$E_1 = 25E_2,\quad G_{12} = G_{13} = 0.5E_2,\quad G_{23} = 0.2E_2,\quad \nu_{12} = 0.25, \alpha_2/\alpha_1 = 3$$

The shear correction coefficients $K_{44}^2 = K_{55}^2$ for the first-order theory are taken to be 5/6. The notation C–F, for example, means that the edge, $x_1 = -L/2$ is clamped and $x_1 = L/2$ is free.

The deflected shape of the circular cylindrical shell has been presented in Figs 1 and 2 for uniform and in Figs 3–5 for linearly varying temperature field through the thickness, where the effect of boundary condition and number of layers is exerted. It is observed that the C–F boundary condition gives the maximum thermal response for all lamination schemes. It is interesting to see that for hinged-hinged circular cylindrical shells, the thermal response is negative for 2 layer (0/90) antisymmetric cross-ply shells, while for 3 layer (0/90/0), 4 layer (0/90/...) and 10 layer (0/90/...), the response is positive. For clamped-clamped circular cylindrical shells subjected to uniform thermal loading through the thickness, the symmetric cross-ply stacking sequence gives greater response than antisymmetric cross-ply ones.

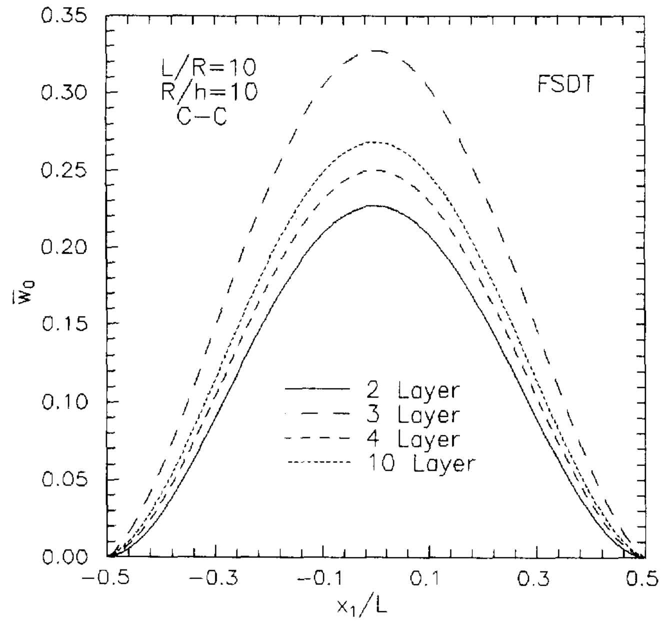

Fig. 1. Deflected shape $\bar{w}_0 = w(x_1, 0, \zeta)h/\alpha_1 \bar{T}_0 R^2$, ($\bar{T}_1 = 0$), along the axial length of a clamped-clamped circular cylindrical shell.

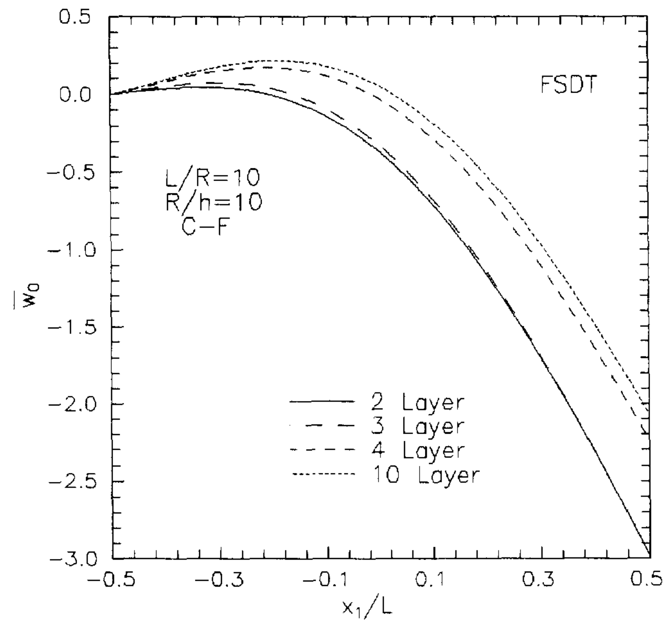

Fig. 2. Deflected shape $\bar{w}_0 = w(x_1, 0, \zeta)h/\alpha_1 \bar{T}_0 R^2$, ($\bar{T}_1 = 0$), along the axial length of a clamped-free circular cylindrical shell.

The tables allow one to conclude the following:

1. The state space approach used to provide exact solutions for the thermal response of cross-ply circular cylindrical shell for arbitrary boundary condition has no limitations. It can be used for thick and thin shells and it has been found to be of great computational efficiency.

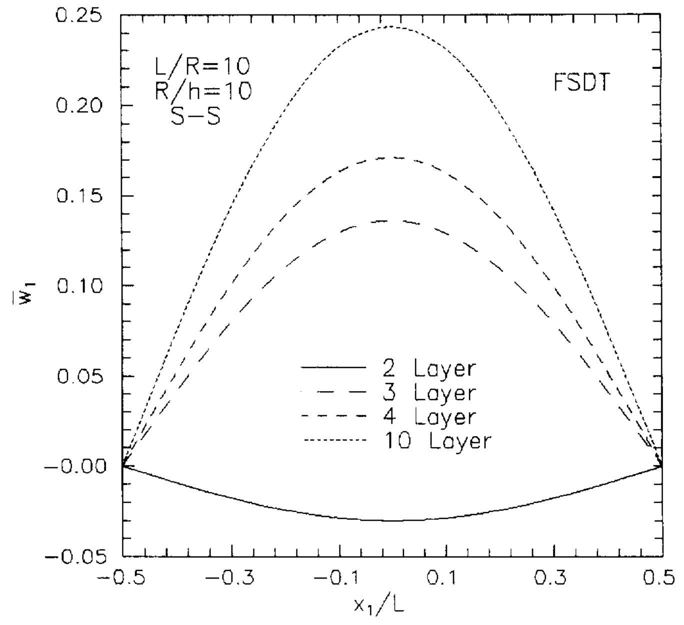

Fig. 3. Deflected shape $\bar{w}_1 = w(x_1, 0, \zeta)/\alpha_1 \bar{T}_1 R^2$, ($\bar{T}_0 = 0$), along the axial length of a hinged-hinged circular cylindrical shell.

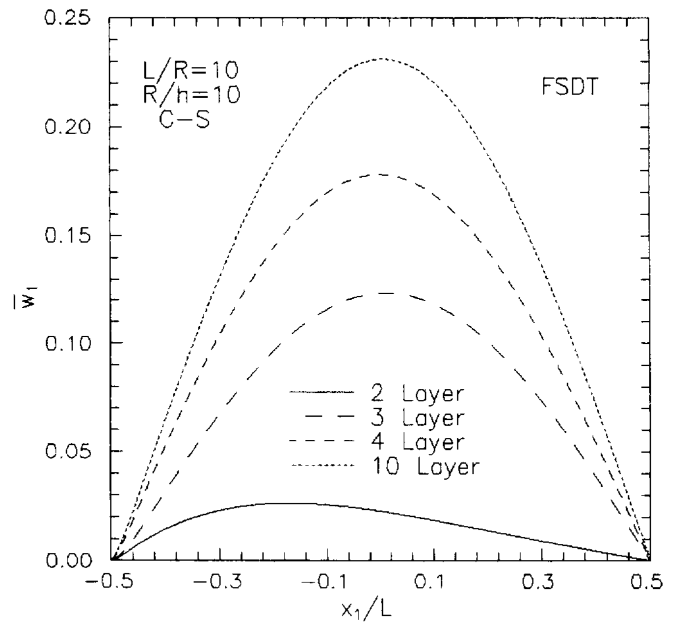

Fig. 4. Deflected shape $\bar{w}_1 = w(x_1, 0, \zeta)/\alpha_1 \bar{T}_1 R^2$, ($\bar{T}_0 = 0$), along the axial length of a clamped-hinged circular cylindrical shell.

2. For a moderately thick circular cylindrical shell, the results for deflections and stresses predicted by HSDT and FSDT are in excellent agreement and close to CST. For thin shells three theories yield the same results because shear deformation has no effect.

Since the three theories have close results, it is preferable for the design engineer to use the CST theory for the analysis of thermoelastic behavior of cross-ply laminated circular

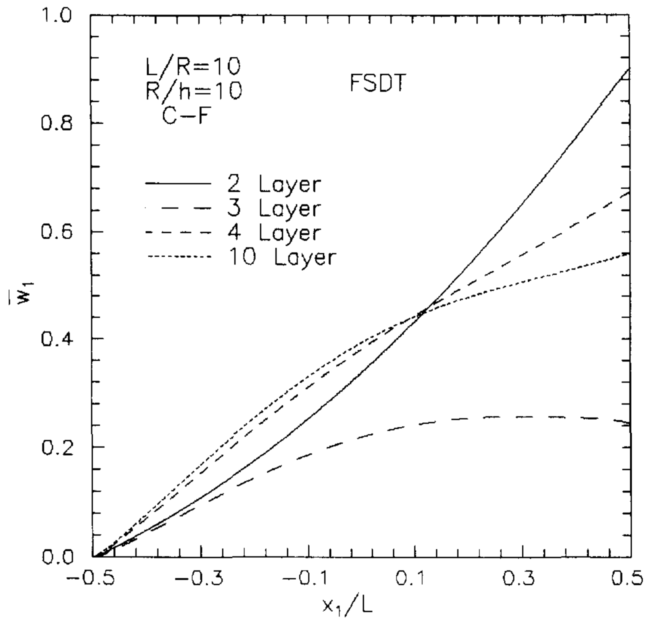

Fig. 5. Deflected shape $\bar{w}_1 = w(x_1, 0, \zeta)/\alpha_1 \bar{T}_1 R^2$, $(\bar{T}_0 = 0)$, along the axial length of a clamped-free circular cylindrical shell.

Table 1. Transverse deflection $\bar{w}_0 = w(0, 0, \zeta)h/\alpha_1 \bar{T}_0 R^2$, $(\bar{T}_1 = 0)$, of cross-ply circular cylindrical shell for all boundary conditions, $L/R = 10$

			$\bar{w}_0$			
Lamination	R/h	Theory	S–S	S–C	C–C	C–F
0/90		HSDT	0.9900	0.6679	0.2269	−0.3689
	10	FSDT	0.9902	0.6683	0.2269	−0.3694
		CST	0.9867	0.6643	0.2248	−0.3692
		HSDT	0.1205	0.0840	0.0338	−0.0551
	100	FSDT	0.1205	0.0840	0.0338	−0.0551
		CST	0.1205	0.0840	0.0338	−0.0551
0/90/0		HSDT	1.1284	0.8210	0.3273	−0.3266
	10	FSDT	1.1284	0.8207	0.3272	−0.3268
		CST	1.1280	0.8146	0.3250	−0.3287
		HSDT	0.1186	0.0874	0.0348	−0.0493
	100	FSDT	0.1186	0.0874	0.0347	−0.0493
		CST	0.1186	0.0874	0.0347	−0.0493

cylindrical shells. The CST theory is more efficient in computation and analysis compared to FSDT and HSDT and has no problems such as ill conditioned matrices during computation. Moreover, in using the state space approach, the CST is easy to handle, since the order of the state vector is 8 compared to 10 for FSDT and 12 for HSDT. Eight eigenvalues with 8 corresponding eigenvectors are needed to evaluate for CST compared to 10 for FSDT and 12 for HSDT.

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Table 2. Axial stress $\bar{\sigma}_1 = \sigma_1(0, 0, h/2)h/\alpha_1 \bar{T}_0 E_2 R$, ($\bar{T}_1 = 0$) of cross-ply circular cylindrical shell for all boundary conditions, $L/R = 10$

Lamination	R/h	Theory	$\bar{\sigma}_1$ S–S	S–C	C–C	C–F
0/90		HSDT	−0.1811	−0.2122	−0.2555	−0.1885
	10	FSDT	−0.1811	−0.2122	−0.2556	−0.1886
		CST	−0.1811	−0.2122	−0.2556	−0.1886
		HSDT	−0.0187	−0.0217	−0.0258	−0.0188
	100	FSDT	−0.0187	−0.0217	−0.0258	−0.0188
		CST	−0.0187	−0.0217	−0.0258	−0.0188
0/90/0		HSDT	0.2147	−0.4342	−1.5310	0.1684
	10	FSDT	0.2147	−0.4343	−1.5310	0.1685
		CST	0.2157	−0.4353	−1.5301	0.1691
		HSDT	0.0111	−0.0518	−0.1583	0.0109
	100	FSDT	0.0111	−0.0518	−0.1583	0.0109
		CST	0.0111	−0.0518	−0.1583	0.0109

Table 3. Circumferential stress $\bar{\sigma}_2 = \sigma_2(0, 0, h/2)h/\alpha_1 \bar{T}_0 E_2 R$, ($\bar{T}_1 = 0$) of cross-ply circular cylindrical shell for all boundary conditions, $L/R = 10$

Lamination	R/h	Theory	$\bar{\sigma}_2$ S–S	S–C	C–C	C–F
0/90		HSDT	0.8303	0.6227	0.3387	−0.0695
	10	FSDT	0.8236	0.6171	0.3343	−0.0720
		CST	0.8405	0.6312	0.3462	−0.0644
		HSDT	0.0269	0.0251	0.0227	0.0151
	100	FSDT	0.0269	0.0251	0.0227	0.0151
		CST	0.0269	0.0251	0.0227	0.0151
0/90/0		HSDT	−0.1254	−0.1465	−0.1807	−0.1980
	10	FSDT	−0.1255	−0.1465	−0.1808	−0.1980
		CST	−0.1251	−0.1466	−0.1808	−0.1982
		HSDT	−0.0176	−0.0184	−0.0196	−0.0185
	100	FSDT	−0.0176	−0.0184	−0.0196	−0.0185
		CST	−0.0176	−0.0184	−0.0196	−0.0185

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